Cascade of Small Nonlinear Oscillations
Triggering Flares

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Abstract. The nonlinear Callebaut-Fourier analysis studies the whole family of higher order terms associated with a single first order wave. Next, it studies the combination of several first order perturbation waves. This family becomes divergent for certain phases if the sum of the amplitudes of the first order terms exceeds a critical limit. This convergence limit has been calculated analytically in some cases and numerically in many cases. Thus a combination of small oscillations may yield local divergences, leading to an explosive situation and instability. In thin 'singular slits' (periodic in space and time) the speeds become very large, theoretically even infinite, creating runaway electrons explaining part of the high energetic particles in flares. The formation of the singular slits results in a fine filamentation of the magnetic field for which much shorter dissipation times apply even in ordinary magnetohydrodynamics (MHD). This triggering mechanism may explain the beginning, the flash and the further decay of flares. The most appropriate regions for this triggering are those where present day reconnection models apply. Moreover, a cascade reaction may occur: sound waves at the solar surface may combine to trigger a bright point, several bright points may combine to trigger a prominence or a solar flare or a coronal mass ejection (CME), each time involving a much larger energy output. An extension for the case when the waves are not parallel is now proposed too.

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1. Introduction

A kind of nonlinear Fourier analysis for systems of nonlinear partial differential equations was developed by one of us, Callebaut (1972). This Callebaut-Fourier analysis was further elaborated for the study of perturbations (oscillations and instabilities) to cases such as plasma waves, gravitational waves and instabilities in hydrodynamic, magnetohydrodynamic (MHD) and even gravitating MHD configurations with boundaries or having infinite extensions in one, two or three directions. See Callebaut (1972), Callebaut and Karugila (2003, 2005, 2006a,b, 2008), Callebaut et al. (2006).

The simple case of cold plasma was investigated in detail by Callebaut & Karugila (2003). The series of higher order terms was obtained, even with explicit analytic coefficients. Thus the convergence of the whole series was studied; we found that when the amplitude of the first order term was more than $e^{-1}$ (37%) of the equilibrium density the series diverged (in density and in velocity). Figure 1 shows a convergent case ($A = 0.3$), Figure 2 shows a divergent one ($A = 0.4$). The divergence occurs in a narrow phase interval ('slit'). Hence this repeats itself periodically in space (every wavelength) and in time (every period). This is a new kind of instability generated from a wave. This divergence-instability "slices" the plasma in slabs of a wavelength thick between two 'divergent slits'. If a magnetic field is present it will similarly be sliced in slabs between singular slits. In those thin singular slits the speeds become very large, theoretically even infinite, creating runaway electrons explaining part of the high energetic particles in flares. Moreover the accelerated ions may create turbulent resistivity in the narrow slits.
Figure 1. The convergent case: Graphs of $\chi = \omega t + kr$ against $n$ (electron number density) for various values of $J$ (the number of terms taken into account) with $n_0$ (the particle density at equilibrium) being scaled to one and $A$ (the amplitude) being put equal to 0.3.

Figure 2. Figure 2. The divergent case: Graphs of $\chi$ against $n$ for various values of $J$ with $n_0$ being scaled to one and the amplitude $A = 0.4$. For divergent series (i.e. when $A > e^{-1} = 0.367879...$) the graphs give some negative values in $n$, which is physically prohibited.

Figure 3. Figure 3. The graphs of total $n$ (electron number density: $n_0 = 1$) against the phase for various orders ($J = 1, 3, 10$ and 100) starting with two initial perturbations with amplitudes $A = 0.3$ and $B = 0.1$. The ratios of frequencies are $\mu = 2$ in (a) and $\mu = 5$ in (b).

divergence occurs, thus destroying the magnetic field very quickly in the thin phase intervals where the divergence or near-divergence occurs.

2. Possible Application to Solar Flares

In a previous paper, Callebaut & Karugila (2006), the idea was mentioned that this divergence (explosion, instability) caused by a wave might explain the baffling behaviour of some plasmas, e.g. in tokamaks or in solar prominences or other magnetic configurations in the solar atmosphere (corona); prominences may remain quiet during weeks, even months (they may move a bit or oscillate), until they suddenly, without apparent reason, burst out in an explosion with an extreme violence (sometimes more than $10^{25}$ J). The flash phase is about 1000 s, although some are shorter than one second, while the further decay may take a full day for a big flare or a coronal mass ejection (CME). Right rom the flash phase there are high energetic electrons and accompanying high frequency waves, X-rays and $\gamma$-rays. Originally our idea to link the divergence of the waves to the outburst of solar flares was a far shot as a perturbing amplitude of the first order term has to be 37 % of the equilibrium amplitude, which is quite large. However, including the pressure decreases the convergence limit. Moreover, we next found a powerful argument supporting our idea: indeed if there are several waves occurring together in a small interval of time each generates its own series of higher order terms and in addition a multitude of mixed terms appear. Thus all these terms contribute and for certain phases their sum may be quite large. In fact it becomes divergent if the sum of all their first order terms is larger than 37 % (or less, taking into account the pressure contribution) of the equilibrium density (Figure 3). This condition is much less stringent than when only one "family" occurs: to have several small or moderate waves reaching together 37% is easier realised (in laboratory and in nature) than a single giant one. These localised explosions (singular slits) "cut" the plasma in narrow slabs so that if a magnetic field is present it may be destroyed at the locations of the divergent phases. Suppose e.g. that the prominence has a thickness $L = 5000$ km, with resistivity $\eta = (\mu \sigma)^{-1} = 100$ m$^2$/s which is 10000 times more than copper. According to customary MHD the characteristic decay time is

$$\tau = \frac{L}{4\pi^2 \eta}$$

and thus of the order of $6 \times 10^9$ s or 200 years, seven orders of magnitude more than the flash! However if the magnetic configuration is perturbed by waves of say 100 m wavelength and the slicing takes place every 10 wavelengths (a reasonable situation in the case that two or more initial waves are present as (roughly) a common multiple of the various wavelengths involved takes over the role of the single wavelength) than the new $\tau = 250$ s. Of course some waves have much smaller wavelengths. This may explain the cases where very short pulses from flares are observed. However, wavelengths smaller than a meter will give a decay time in the range of milliseconds and less. Then the opposite problem might appear: too short dissipation times in comparison with the observed values.
3. Combination with Reconnection Theory

The various reconnection theories to explain the outburst of a flare use basically a compression of plasma and magnetic field followed by some annihilation of part of the field followed by reconnection of magnetic field lines. That is a valuable approach. However, the time for decay turns out $10^6$ to $10^9$ times too long. The usual argument to save the theory is that turbulent resistivity occurs, which may be up to a factor $10^6$ larger than the normal one. However, how is this turbulence generated? Once the flare starts one can invoke the turbulence but not before it. The approach given above is perfectly compatible with reconnection and allows reasonable decay times. Afterwards turbulence may set in. Moreover, high velocity electrons (and thus X-rays and γ-rays) are created right from the beginning as happens in the observations.

4. Conclusions

The present approach may be the beginning of the solution to the longstanding huge problem of the reconnection theory for solar flares and some related phenomena: the discrepancy of a factor $10^6$ to $10^9$ in dissipation time (or in resistivity). Reconnection theory has a lot of good points and removing its main obstacle without affecting its mechanism by only adding a trigger seems very advantageous. In addition our approach allows highly energetic particles from the beginning, which is not obvious when using just ohmic dissipation. Moreover, a certain cascade mechanism may occur: several acoustic waves at the photosphere may combine to create instability of a larger feature, e.g. a bright point. The shocks and waves emitted by one or several bright points may combine to render a prominence unstable and to cause a flare and/or a CME.

Clearly the present nonlinear theory combining several waves needs still further elaboration: extension to non-periodic waves, detailed study of the dissipation of magnetic energy in the narrow slits and in the slabs between them, probability of occurrence, etc.

References